ISSN: 2277-9655
Impact Factor: 1.852

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

Reparametrization of Interval Bézier Curve on Rectangular and Circular Domain

Megha Jain[*1], Pranjali Arondekar[2], Ashok Ganguly[3]
[*1]Department of Applied Mathematics and Computational Science, Shri G.S.Institute of Technology and Science, 23, M.Visvesvaraya Marg, Indore-03, India
[2]Department of Mathematics and statistics, Medi Caps Inst. Of Tech. and Management, Indore, India
[3]Department of Applied Mathematics and Computational Science, Shri G.S.Institute of Technology and Science, 23, M.Visvesvaraya Marg, Indore-03, India
megha.2672010@gmail.com

Abstract

Interval Bezier curve are new representation forms of parametric curves that can embody a complete description of coefficient errors. Using this new representation, the problem of lack of robustness in all state-of-the art CAD systems can be largely overcome. In this paper we discuss this concept to form a new curve over rectangular and circular domain such that its parameter varies in an arbitrary range [a, b] instead of standard parameter [0, 1]. Where a and b are real and, we also want that curve gets generated within the given error tolerance limit.

Keywords: Interval Bézier curve, rectangular domain, circular domain.

Introduction

An interval Bézier curve is a Bézier curve whose control points are rectangles (the sides of which are parallel to coordinate axis) in a plane. Such a representation of parametric curves can account for error tolerances. Based upon the interval representation of parametric curves and surfaces, robust algorithms for many geometric operations such as curve/ curve intersection were proposed [6]. The series of works by Hu et. al. indicate that using interval arithmetic will substantially increase the numerical stability in geometric computations and thus enhance the robustness of current CAD/CAM systems.

The most of the approximation schemes guarantee that an approximation will satisfy a prescribed tolerance; once this has been achieved none proposes to carry detailed information on the approximation error forward to subsequent applications in other systems.

In this paper we use the concept of David Solemon [7] in reparametrizing a Bézier curve such that its parameter varies in an arbitrary range [a, b] we obtain a new curve denoted by $P_{ab}(t)$ which is simply the original curve with a different parameter $P_{ab}(t) = P\left(\frac{t-a}{b-a}\right)$. It helps to calculate a curve $Q(t)$ defined on an arbitrary part of $P(t)$, where $0 \le t \le 1$. Here the basic approach is to define a new curve $Q(t)$ as $P\left(\left[b-a\right]t+a\right)$ and express the control points Q_i of $Q(t)$ in terms of the control points P_i and a and b.

In the present paper our aim is to take the above concept in the context of rectangular and circular interval Bézier curve. We will introduce the concept of reparametrization by making use of the Bézier curve on rectangular and circular domain.

We organize the paper as follows, in section 2, we propose the reparametrization concept on interval Bézier curve on rectangular domain, the same concept on circular domain is provided in section 3, and we conclude the paper in section 4.

For the software implementation to represent reparametrization in interval form of Bézier curve, we use the mathematical software MATLAB.

Interval Bézier Curve on Rectangular Domain

Basics of Rectangular Domain.

Interval Bézier curves were first studied by Sederberg and Farouki [5]. They introduced a new representation form of parametric curves *viz* the interval Bézier curve that can transfer a complete description of approximation errors along with the curve to applications in other systems.

Vector-valued interval $[P]$ in the most general terms is defined as any compact set of points (x, y) in two dimensions as tensor products of scalar intervals

$$[p]=[a,b]\times[c,d]=\{(x,y)\,|\,x\in[a,b]\ and\ \ y\in[c,d]\}$$

.Such vector-valued intervals are clearly just rectangular regions in plane [3]. The addition of such sets is given by the Minkowski sum

$$[p_1]+[p_2]=\{(x_1+x_2,y_1+y_2)\,|\,(x_1,y_1)\in[p_1]\ and\ (x_2,y_2)\in[p_2]\} \quad \textbf{(2.1)}$$

An interval Bézier curve with rectangular vector interval $[p_i]$ is written in the form

$$[p](t)=\sum_{i=0}^{n}[p_i]B_{n,i}(t) \qquad 0\le t\le 1 \quad \textbf{(2.2)}$$

where $B_{n,i}(t)$'s are the Bernstein polynomials and $[p_i]=([a_i,b_i],[c_i,d_i])$ are rectangular control points .i.e. to define interval Bézier curve on rectangular domain we have to define interval range for both x and y direction for each control point.

Reparametrization on Rectangular Domain

As shown by solemon [7], we will reparameterize $[P](t)\ as\ [Q](t)$ in the new parameter range [a, b] instead of standard parameter range [0, 1]. This approach will lead to a new curve on rectangular domain as $[P]_{ab}(t)=[P]\left(\frac{t-a}{b-a}\right)$, where it has to satisfy the condition $[Q](0)=[P](a)\ and\ [Q](1)=[P](b)$ on rectangular domain.

So,

$$[Q](t)=[P]([b-a]t+a)$$

$$\sum_{i=0}^{n}[Q]_i B_{n,i}(t)=\sum_{i=0}^{n}[P]_i\,B_{n,i}([b-a]t+a)$$

By applying interval operation mentioned in [7], and by comparing the coefficient of various powers of t, we get

$$[Q_0]=(1-a)^3[P_0]+3(a-1)^2a[P_1]+3(1-a)a^2[P_2]+a^3[P_3]$$

$$[Q_1]=(a-1)^2(1-b)[P_0]+(a-1)(-2a-b+3ab)[P_1]+a(a+2b-3ab)[P_2]+a^2b[P_3]$$

$$[Q_2]=(1-a)(-1+b)^2[P_0]+(b-1)(-a-2b+3ab)[P_1]+b(2a+b-3ab)[P_2]+ab^2[P_3]$$

$$[Q_3]=(1-b)^3[P_0]+3(b-1)^2b[P_1]+3(1-b)b^2[P_2]+b^3[P_3]$$

Figure 1, shows the reparametrization of the interval Bézier curve on rectangular domain in the range [1,2].

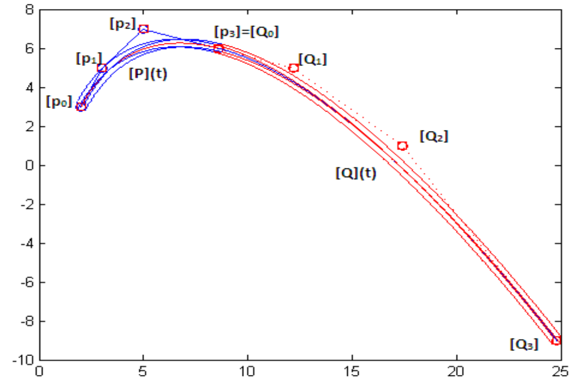

Figure 1. Reparametrization of Bézier curve on rectangular domain

For the curve in above figure, the calculated values of control points of reparameterized interval curve is shown in the table 1, with a=1, b=2. Here, $[P_i]$ denotes the rectangular control points for the original curve and $[Q_i]$ denotes the reparameterized control points of the new curve with specified errors.

Table.1

i	Control points of $[P](t)$ $[P_i]$	Reparametrized control points of $[Q](t)$ $[Q_i]$
0	$[1.8,2.2]\times[2.8,3.2]$	$[8.3,8.7]\times[5.8,6.2]$
1	$[2.8,3.2]\times[4.8,5.2]$	$[11.8,12.2]\times[4.8,5.2]$
2	$[4.8,5.2]\times[6.8,7.2]$	$[16.8,17.2]\times[0.8,1.2]$
3	$[8.3,8.7]\times[5.8,6.2]$	$[23.8,24.2]\times[-9.2,-8.8]$

Interval Bézier curve on Circular Domain

Basics of Circular Interval

Disk or circular interval is defined as follows ([2], [4]):

$$(P)=P_r(c_x,c_y)=\{(x,y)\in R^2\,|\,(x-c_x)^2+(y-c_y)^2\le r^2\}$$

$$=\{(x,y)\in R^2\,|\,(x-y)\in(c_x,c_y)+r(\cos\theta,\sin\theta),0\le\theta\le 2\pi\}=(c,r)$$

Where c is the center, (c_x,c_y) will be considered as given point and r is the radius of circular interval (P). Thus to capture the uncertainty characteristics of parameters, we consider the real co-ordinates of the given points and nearby values in a particular neighborhood having radius equal to the allowable error tolerance.

Circular interval linear operation is defined as:

$$(P_1)+(P_2)=(c_1,r_1)+(c_2,r_2)=(c_1+c_2,r_1+r_2)$$ **(3.1)**

with width $(P)=2r$.

Reparameterization on Circular Domain

Using the circular interval defined by (3.1) as controlling polygon, interval Bézier curve on circular domain is defined as follows:

$$(P)(t)=\sum_{i=1}^{n+1}(P_i)B_{n,i}(t)$$

Where

$$(P_i)=(c_{x_i},c_{y_i})+\varepsilon_i(cos\theta,\sin\theta)$$

For $0\le\theta\le 2\pi$ **(3.2)**

$(c_{x_i},c_{y_i})'s$ The given polygon points, with allowable error ε_i, $B_{n,i}(t)$'s the Bernstein Polynomials [4]. By using solemon [7] concept we will reparameterize $(p)(t)$ as $(Q)(t)$ in the new parameter range $[a,b]$ instead of standard parameter range $[0,1]$.

This will be denoted by $(P)_{ab}(t)=(P)\left(\frac{t-a}{b-a}\right)$ where it has to satisfy the condition $(Q)(0)=(P)(a)$ *and* $(Q)(1)=(P)(b)$ on circular domain.

So,

$$(Q)(t)=(P)([b-a]t+a)$$

$$\sum_{i=0}^{n}(Q_i)B_{n,i}(t)=\sum_{i=0}^{n}(P_i)B_{n,i}([b-a]t+a)$$

By applying interval operation mentioned in [8], and by comparing the coefficient of various powers of t, we get

$$(Q_0)=(1-a)^3(P_0)+3(a-1)^2a(P_1)+3(1-a)a^2(P_2)+a^3(P_3)$$

$$(Q_1)=(a-1)^2(1-b)(P_0)+(a-1)(-2a-b+3ab)(P_1)+a(a+2b-3ab)(P_2)+a^2b(P_3)$$

$$(Q_2)=(1-a)(-1+b)^2(P_0)+(b-1)(-a-2b+3ab)(P_1)+b(2a+b-3ab)(P_2)+ab^2(P_3)$$

$$(Q_3)=(1-b)^3(P_0)+3(b-1)^2b(P_1)+3(1-b)b^2(P_2)+b^3(P_3)$$

Figure 2, shows the reparametrization of the interval Bézier curve on circular domain in the range [1, 2].

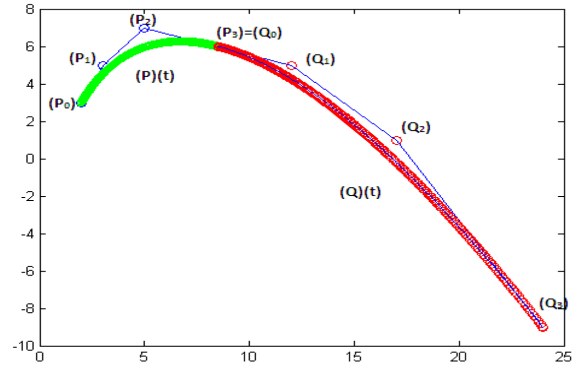

Figure 2. Reparametrization of Bézier curve on circular domain

Conclusion

In this paper, we define a curve in the interval form on rectangular and circular domain for an arbitrary range, we proposed efficient examples to prove this. The result nor only reflects the potential curve in the figures, but also provides an error measures in interval Bézier curve.

References

[1] Arondekar Pranjali, “A Mathematical study of some aspects of Bézier and B-spline curves related to CAGD”, Ph.D thesis, DAVV Indore, 2010.
[2] Chen .F and Yang Wu, “Degree reduction of disk Bézier curves”, computer aided geometric design 21,263-280, 2004.
[3] Ganguly A. and Arondekar P, “ A study related to interval B-spline Curve”, Far East Journal of Applied Mathematics, Vol 34,No.2,2009 pp-151-167.
[4] Lin Q and Rokne J G, “Disk Bezier curve” [J], computer aided geometric design, 15(7):721-732, 1998.
[5] Sederberg TW, Farouki RT, “Approximation by interval Bézier curves”, IEEE comput.Graph Appl., 15(2):87-95, 1992.
[6] Shen.G and Patrikalakis N.M, “Numerical and geometrical properties of interval B-splines” International journal of shape modeling, 4(1/2):35-62, 1998.
[7] Solemon David, “Curves and surfaces for computer graphics”, Springer-Verlag, 2006
[8] Tuohy S.T., Maekawa.T and Patrikalakis.N.M, “Approximation of measured data with interval B-splines”, Computer –Aided Design 29(11):791-799, 1997.